ORDER REDUCTION OF A LARGE-SCALE INDEX-2 DAE MODEL

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Differential algebraic equation (DAE) models of real processes can consist of hundreds of differential equations and thousands of algebraic equations. Order reduction of these models can give insight into the model structure, analysis of dynamic performance, and enable storage and retrieval of optimal control trajectories. Another reason for reduction of the order is to efficiently solve the DAE model in simulation and control applications. However, this last reason is not always valid. Order reduction destroys sparsity in favour of a reduced number of equations and variables. If the sparsity of the original system is exploited in the calculations then there may be little or no numerical advantage of solving a reduced order model.

Model reduction approaches such as balanced covariance matrices (BCM) and proper orthogonal decomposition (POD) have been developed to optimally reduce the number of differential states for nonlinear models. Although effective for differential state reduction, these model reduction approaches do not reduce the number of algebraic equations. Because the algebraic equations often greatly outnumber the differential states, significant order reduction of the overall model is not achieved by POD and BCM. Physical significance of the variables is lost through model reduction. In many applications it is often desirable or required for a reduced model to retain physical significance of the original variables.

Additional approaches have been suggested for DAE model reduction, but they generally suffer from poor scaling to large scale problems or extensive model configuration. The proposed technique in this work has the advantage of good scaling to large scale problems. An added advantage is that the physical significance of the algebraic equations is retained. A major focus of this work is also in making the model reduction approach adaptive, thereby making DAE model reduction accessible to the non-expert user.

Because algebraic equations often greatly outnumber the differential equations, reduction of the differential equations often does little to reduce the overall order (and also computational time) of a DAE model. Efficient algebraic reduction is a major obstacle to overall model reduction of DAEs.

The principal contribution of this work is to demonstrate model reduction of a large-scale DAE. An index-2 DAE model of a 22-stage, 5-component distillation is selected as a test case for model reduction. The index of a DAE refers to the number of times the algebraic equations must be differentiated to transform the model into an ODE. Only a fraction of the equations are index-2 with the majority being index-1. The index-2 equations are the 22 bubble point temperature equations, with 1 in each of the distillation stages. The model is reduced in three steps. The first step involves model reduction at the most basic level. Each stream (or accumulation) includes pressure, temperature, mole fractions, mass fractions, concentrations, molar flow rate (or moles), mass flow rate (or mass), volumetric flow rate (or volume), density, and enthalpy. Pressure, temperature, mole fractions, and molar flow rate (or moles) are used to uniquely specify the state of the mixture. All other variables are solved explicitly as a function of these variables and can be removed from the implicit set. Another reduction in variables is gained from the object-oriented framework. Instead of defining connection equations, two connecting streams can be merged into one stream object. The explicit transformation and stream merging reduces the model size from 3250 to 353 variables. The 353 variable model consists of 107 differential variables and 246 algebraic variables. The index-2 DAE is converted to index-1 form by differentiating the index-2 algebraic equations. The differentiated bubble point temperature equations are used to remove the temperature derivative in the energy balance. By removing the temperature derivatives, the index-2 equations become index-1 equations. In index-1 form, the algebraic equations are explicitly solved at each function call, thereby removing the 246 algebraic variables from the implicit set. The remaining differential variables are reduced with POD to 8 ODEs. The 8 state model has excellent agreement with the full 353 state model in both dynamic response and gain.

Even though the model is drastically reduced in size, no computational advantage for simulation is achieved. The 353 state model step test simulation required 41 CPU seconds on a 2 GHz Celeron processor, running FORTRAN. Based on LINPACK benchmarks for processor MFLOPS (million floating point operations per second), the step test required approximately 30 billion floating point operations. Each reduced model step response was generated in MATLAB, requiring approximately 31 billion floating point operations.

There are several reasons that the reduced model does not substantially reduce the computational burden. The primary reason is that all the equation residuals must still be computed to form the reduced order model equation residuals. The calculation of equation residuals requires 22 billion floating point operations for both the full and reduced models. This is an irreducible overhead, regardless of model size. With efficient DAE solvers, such as DASPK, no computational advantage is gained by solving the algebraic equations explicitly at each residual evaluation. An amount of computational overhead is also added in the index-2 to index-1 transformation.

Clearly, model reduction is not computationally advantageous purely for simulation. Fortunately, there are a variety of other justifications for DAE model reduction. One reason is the insight that model reduction provides. Even though the original model has thousands of variables, there are only 8 degrees of freedom that control the dynamic response. Another reason is for off-line storage and retrieval of control solutions for efficient on-line implementation. Off-line storage and retrieval of control solutions is more efficient for smaller models, with an upper limit of about 100 states. Model reduction is an important enabling step in achieving computationally feasible large-scale DAE model-based control solutions.