In Situ Adaptive Tabulation for Real-Time Control

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Outline

- Model reduction and computational reduction
- Introduction to ISAT
- ISAT theory
- Application #1: Combined Approach
- Application #2: ISAT vs. Neural Nets
- Conclusions

Model Reduction

- Optimally reduce the number of model variables
- Linear combination of states that retain the most important dynamics
- Methods
  - Proper Orthogonal Decomposition (or PCA)
  - Balanced Covariance Matrices

Model Reduction

Original ODE model

$$\boxed{\begin{aligned}\dot{x} &= f(x,u)\\ y &= h(x)\end{aligned}} \qquad (1)$$

Determine a similarity transform to optimally reduce the model states

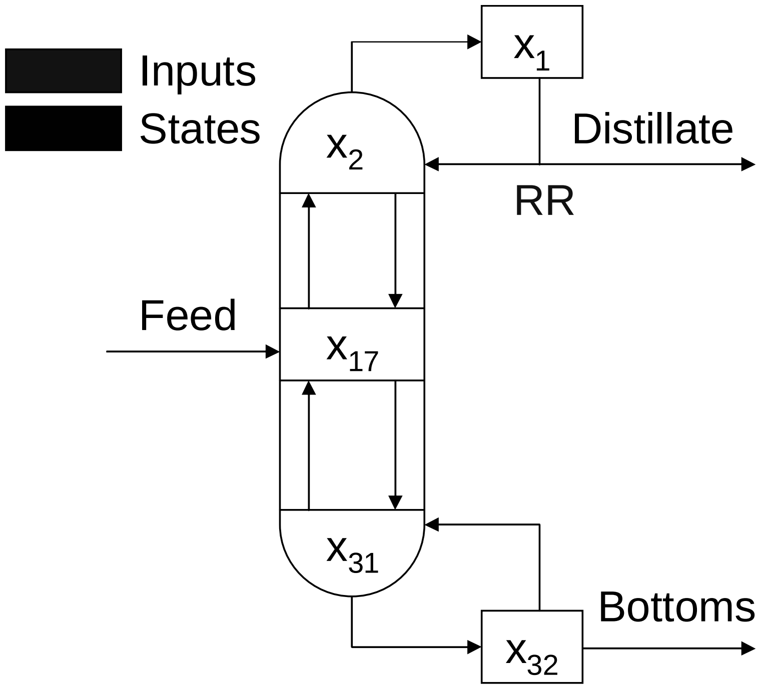
$$\boxed{\begin{aligned}\dot{x} &= (T^{-1}T)\,f(T^{-1}(Tx),u)\\ y &= h(T^{-1}(Tx))\end{aligned}} \qquad (2a)$$

Transformed states

$$\bar{x} = Tx \qquad \boxed{\begin{aligned}\dot{\bar{x}} &= \bar{f}(\bar{x},u)\\ y &= \bar{h}(\bar{x})\end{aligned}} \qquad (2b)$$

Model Reduction

Binary distillation model reduction shows the relative weighting of the 32 original states in the top 3 transformed states.



$$\begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \\ \bar{x}_3 \end{bmatrix} = \begin{bmatrix} 9.1 & \cdots & 0.015 \\ 49.5 & \cdots & -0.060 \\ -4.9 & \cdots & -0.202 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{32} \end{bmatrix} \quad (3)$$

Model Reduction

<u>Truncation</u>

$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \\ \dot{\bar{x}}_4 \\ \vdots \\ \dot{\bar{x}}_{32} \end{bmatrix} = \begin{bmatrix} \bar{f}_1(\bar{x},u) \\ \bar{f}_2(\bar{x},u) \\ \bar{f}_3(\bar{x},u) \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

<u>Residualization</u>

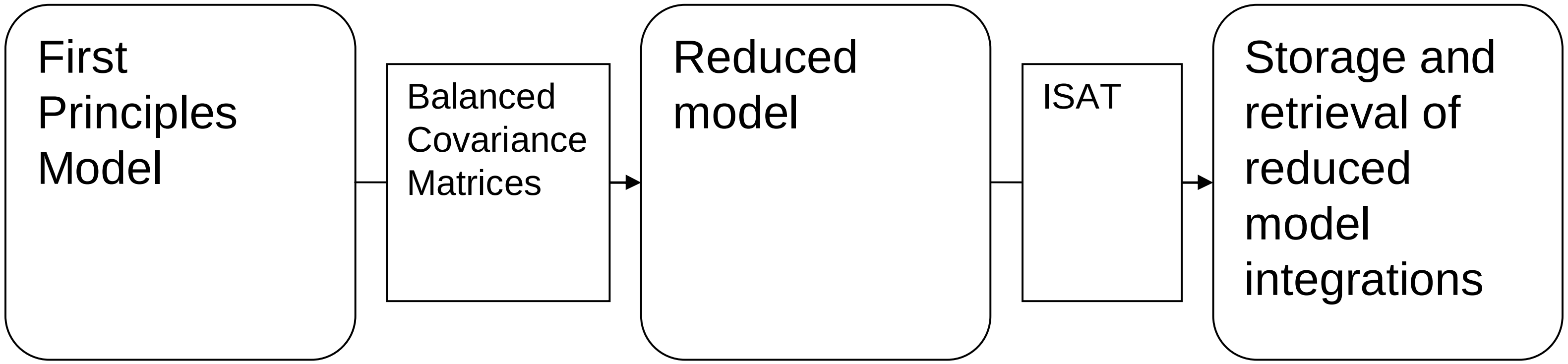
$$\begin{bmatrix} \dot{\bar{x}}_1 \\ \dot{\bar{x}}_2 \\ \dot{\bar{x}}_3 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} \bar{f}_1(\bar{x},u) \\ \bar{f}_2(\bar{x},u) \\ \bar{f}_3(\bar{x},u) \\ \bar{f}_4(\bar{x},u) \\ \vdots \\ \bar{f}_{32}(\bar{x},u) \end{bmatrix}$$

Computational Reduction

- Retain all the of dynamics
- Storage and retrieval to reduce the computational cost
- Methods
  - Artificial neural networks
  - *In situ* adaptive tabulation (ISAT)

Combined Approach

- Combined approach for NMPC
  - Model reduction first
  - Computational reduction second



ISAT Introduction

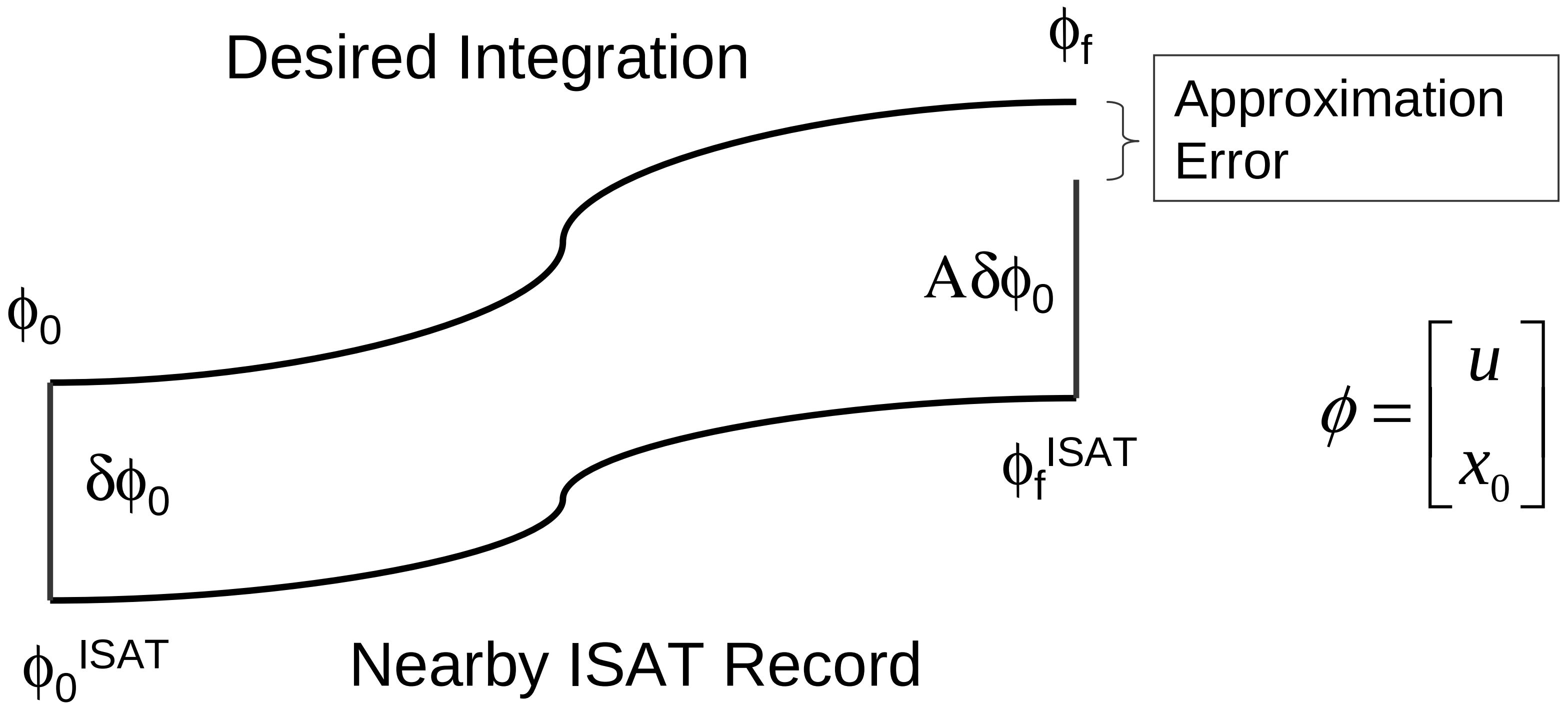
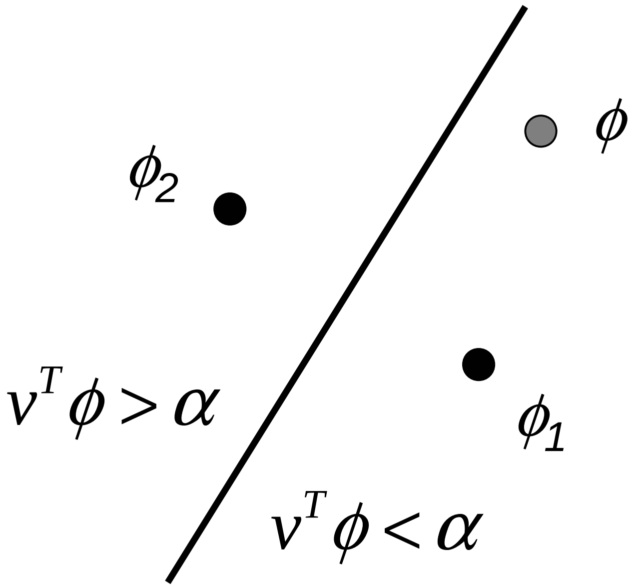


Fig. 1. Approximation of the desired integration final state with a nearby ISAT record.

ISAT Search

- Binary Tree Architecture
  - Search times are *O(log$_2$(N))* compared with *O(N)* for a sequential search



$$v = \phi_2 - \phi_1$$

$$\alpha = v^T\left(\frac{\phi_2 + \phi_1}{2}\right)$$

Binary Trees

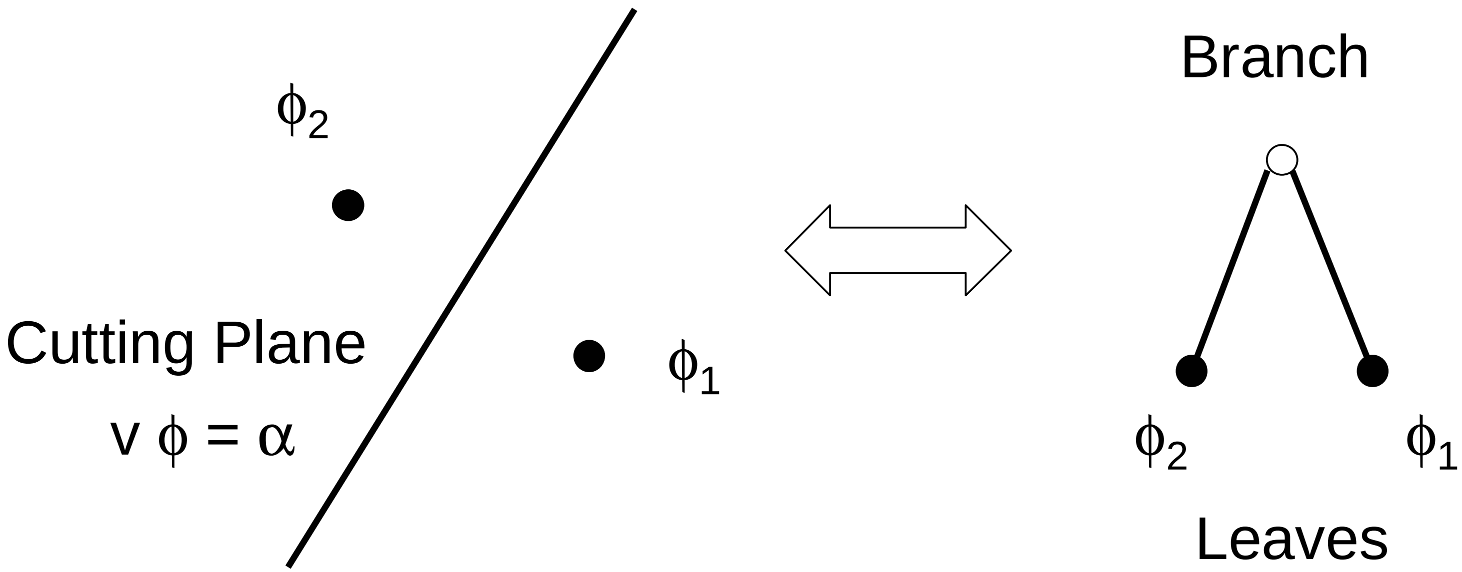


Fig. 2. An illustration of the binary tree structure in the cutting plane format (on the left) and the tree format (on the right).

Binary Tree Growth

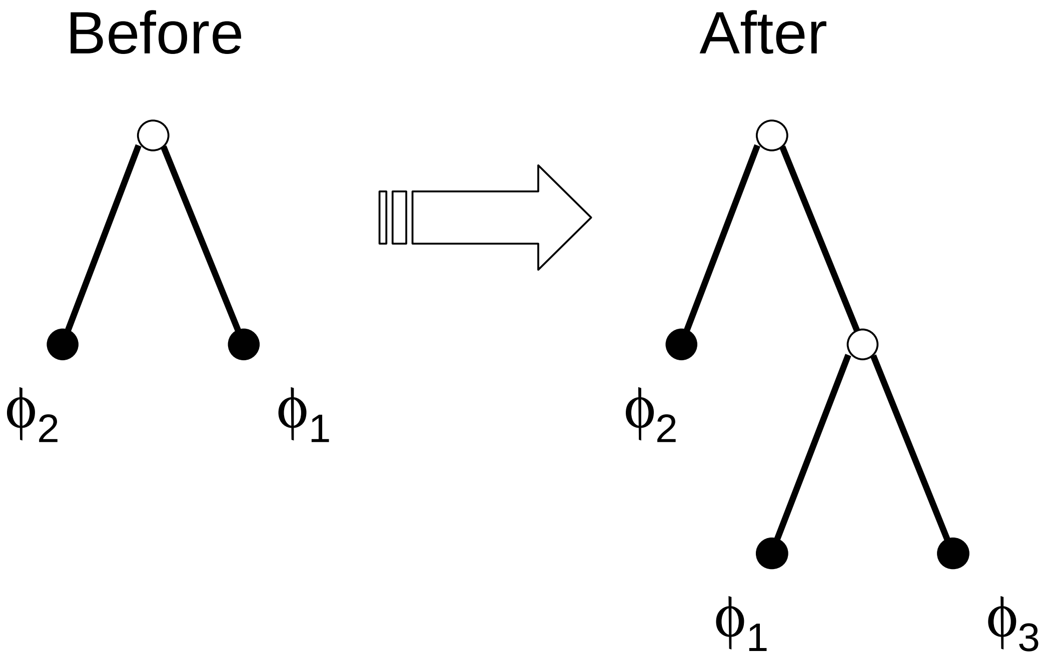


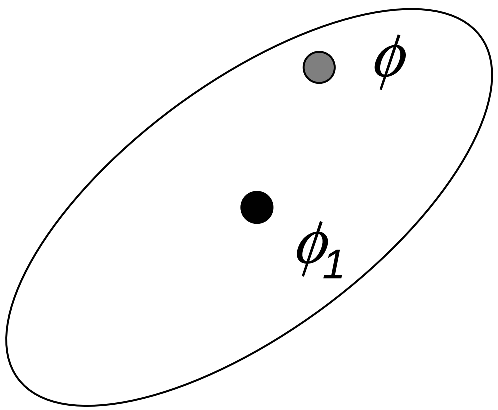
Fig. 3. Binary tree growth. A tree with one branch and two leaves is grown to include another leaf.

Binary Trees

- To increase the accuracy of the binary tree search, multiple binary trees are searched.
- This increases the probability of finding a better record.
- Number of binary trees is a tuning parameter that balances search speed with search accuracy.

ISAT Integration

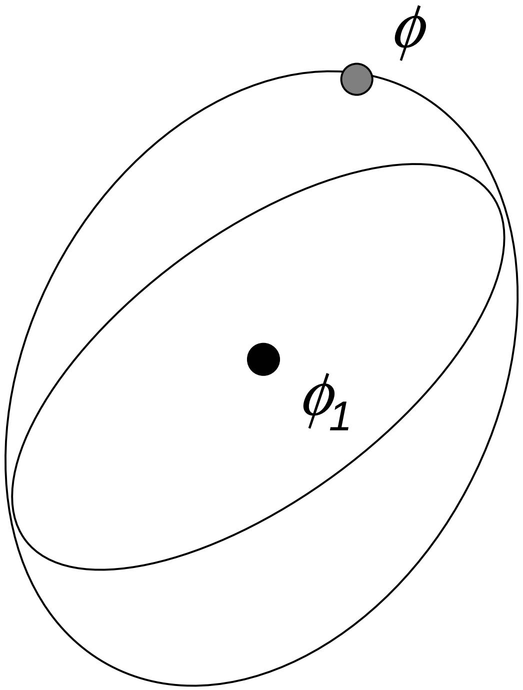
- Scenario #1: Inside the region of accuracy



$$\left(\phi-\phi_1\right)^T M\left(\phi-\phi_1\right)\leq \varepsilon_{tol}$$

ISAT Integration

- Scenario #2: Outside the region of accuracy but within the error tolerance

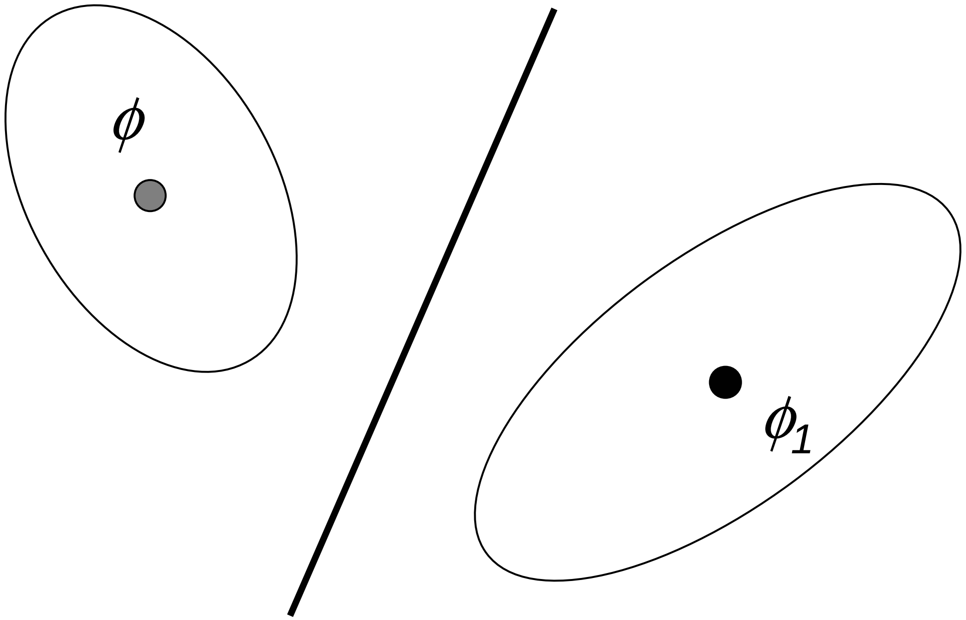


$$\left(\phi-\phi_1\right)^T M\left(\phi-\phi_1\right)>\varepsilon_{tol}$$

Compute M_{new} so that the new region is a symmetric, minimum volume ellipsoid that includes ϕ

ISAT Integration

- Scenario #3: Outside the region of accuracy and outside the error tolerance



Define cutting plane

$$v = \phi - \phi_1$$

$$\alpha = v^T \left(\frac{\phi + \phi_1}{2} \right)$$

Find a conservative estimate for the region of accuracy around ϕ

Application #1: Binary Distillation

Fig. 4. Model and computational reduction flowchart.

Closed-loop Response

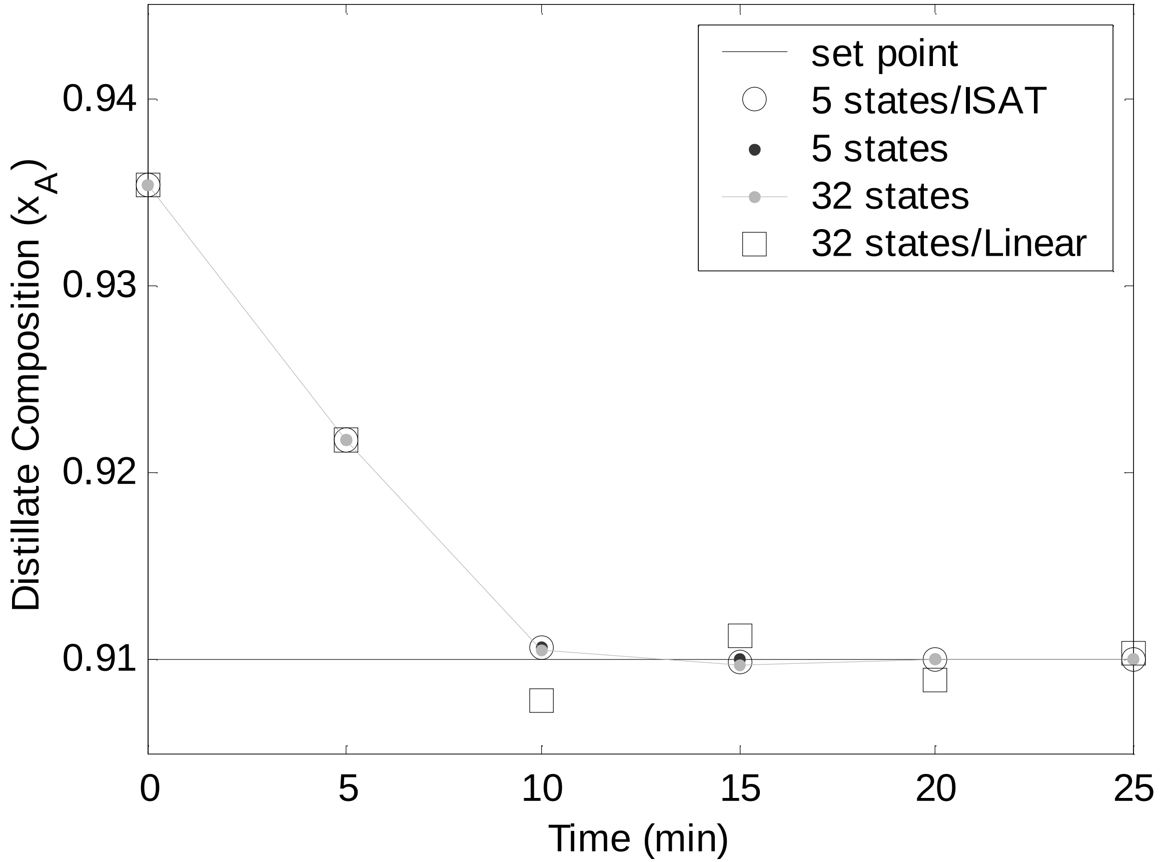


Fig. 5. Closed loop response comparison for nonlinear MPC with ISAT with 5 states, nonlinear MPC with 5 states, nonlinear MPC with 32 states, and linear MPC.

CPU times

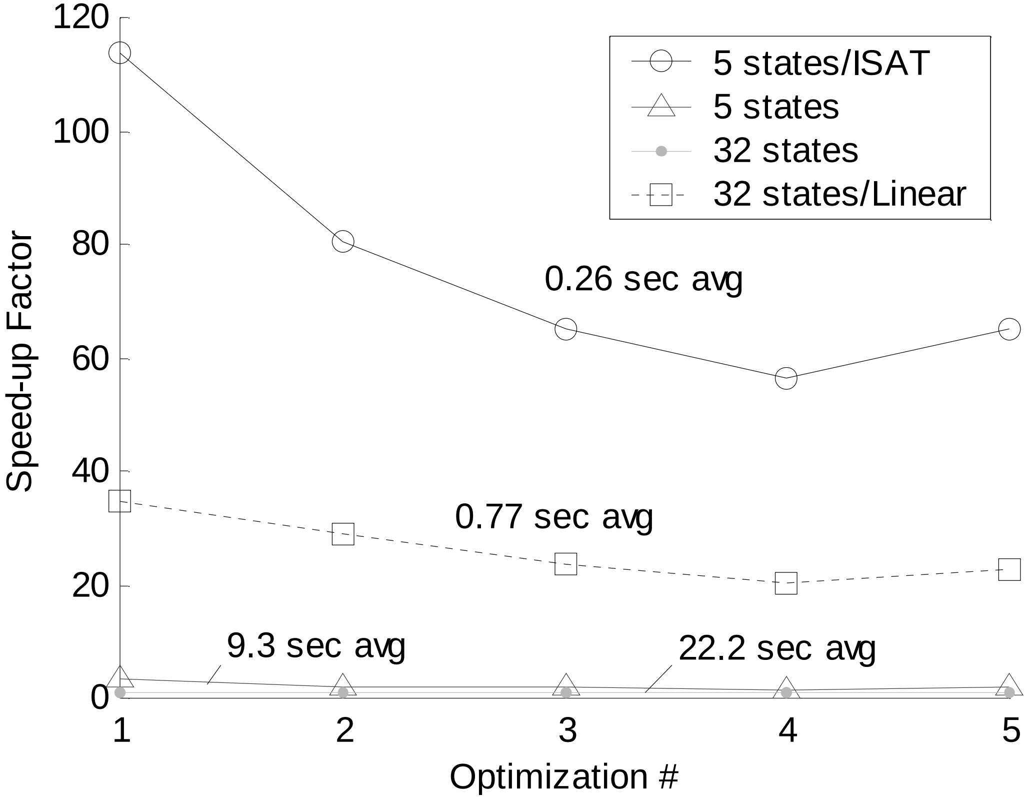


Fig. 6. Speed-up factor for each of the optimizations shown in Fig. 5. The number above each curve indicates the average optimization cpu time on a 2 GHz processor.

Application #2: ISAT vs. neural net

- Dual CSTR model

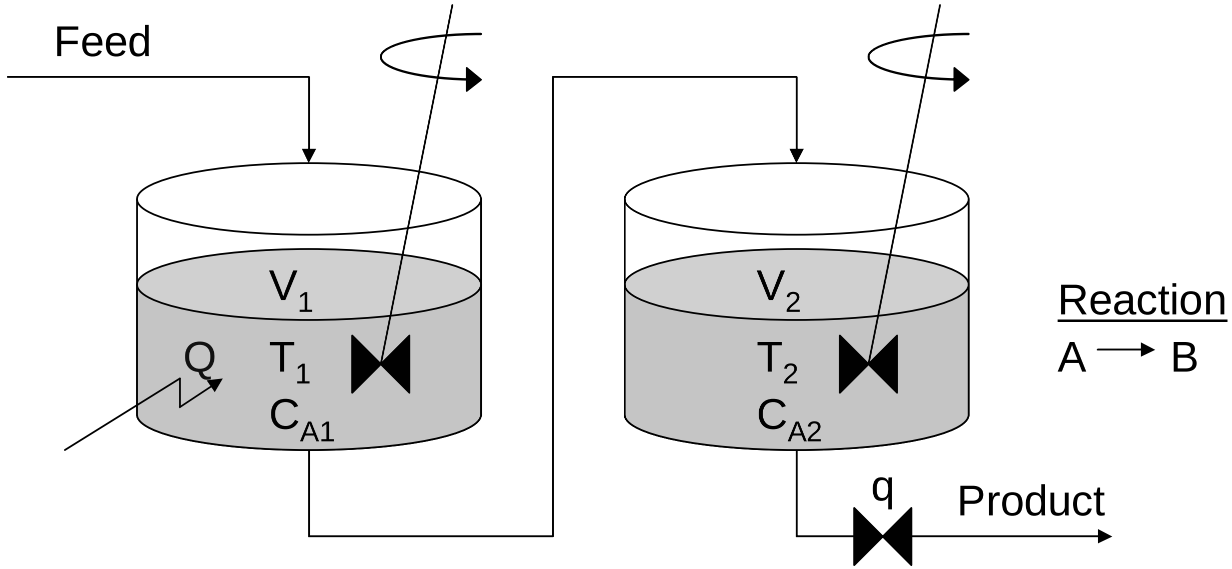


Fig. 7. Diagram of two CSTRs in series with a first order reaction. The manipulated variable is the cooling rate to the first CSTR

Artificial Neural Network

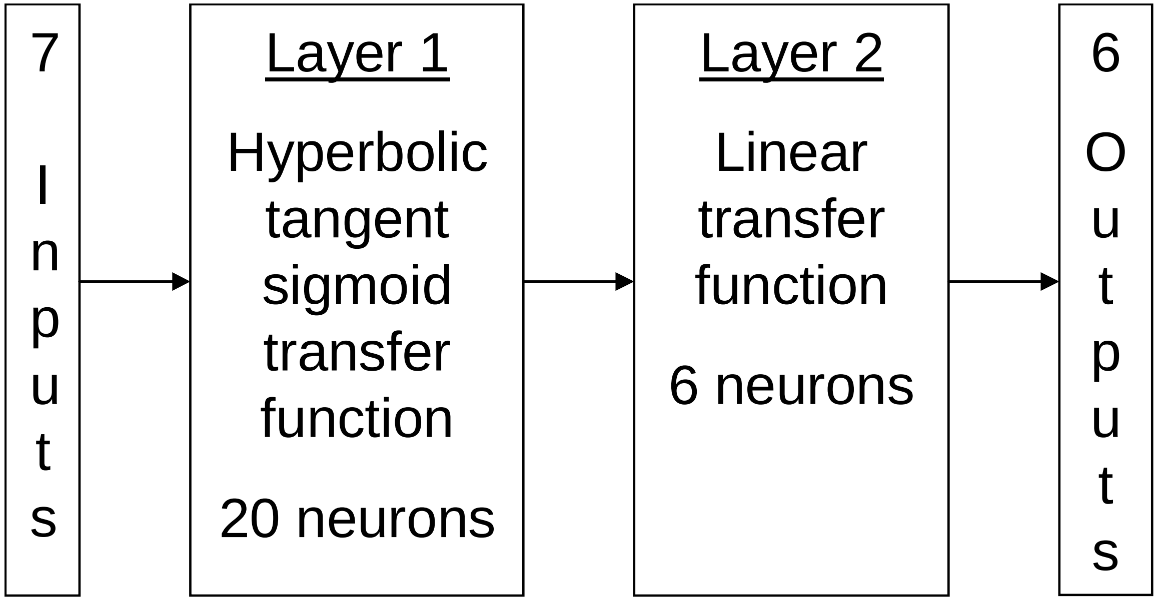


Fig. 8. Neural net with one hidden layer and one output layer. The hidden layer is a hyperbolic tangent function and the output layer is a linear function. This neural net relates 7 inputs to 6 outputs.

Open-loop Response

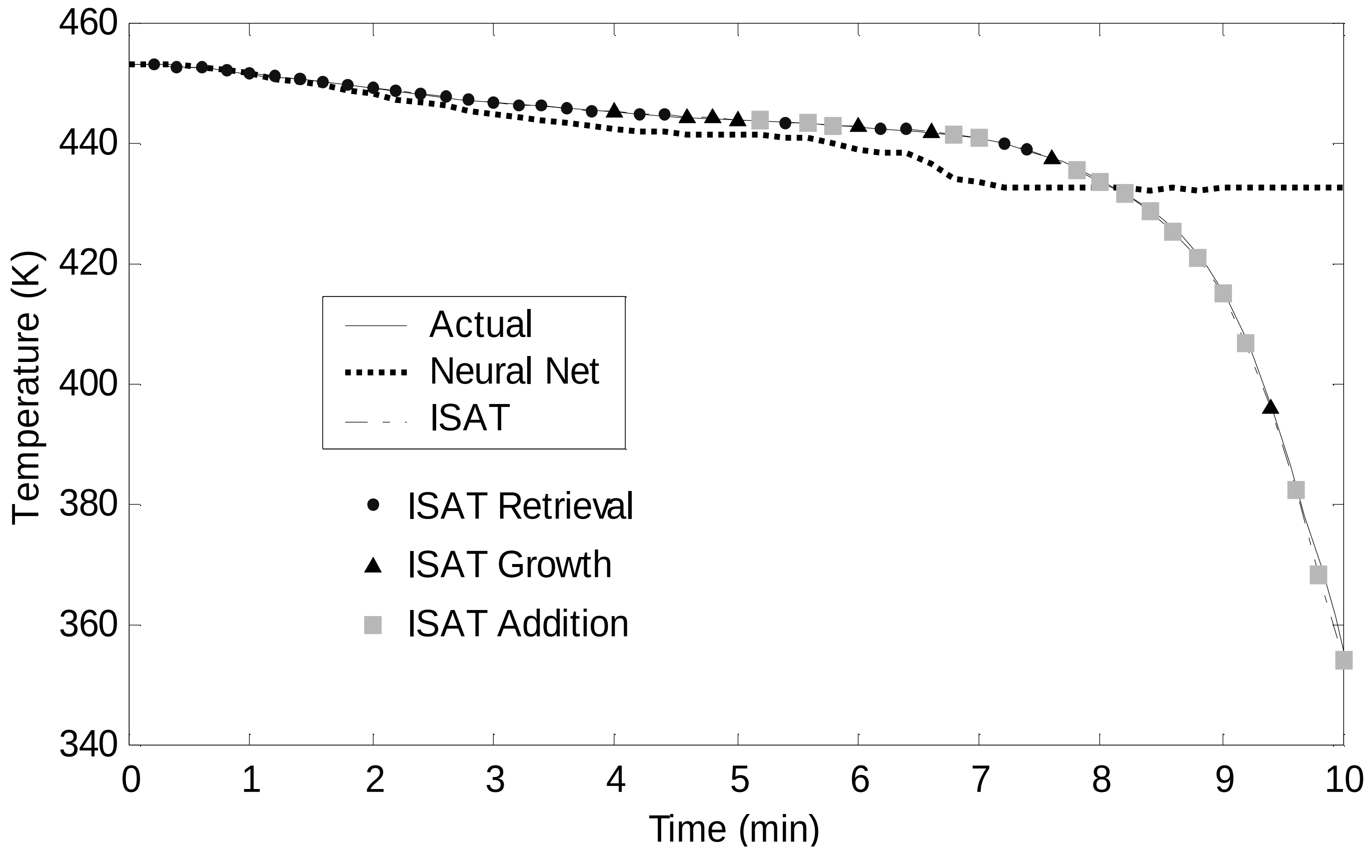


Fig. 9. The error control of ISAT indicates that additional records must be added, thereby avoiding extrapolation error.

Closed-loop Response #1

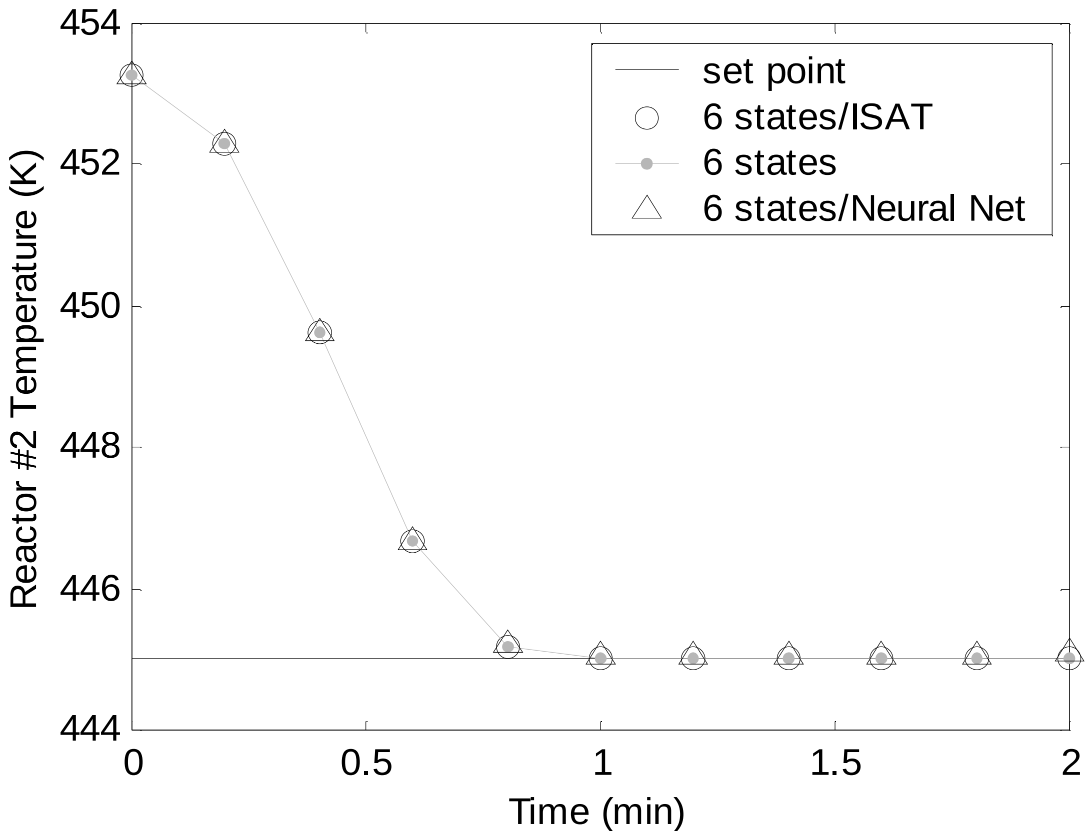


Fig. 10. Small closed loop set point change within the training domain.

Closed-loop Response #2

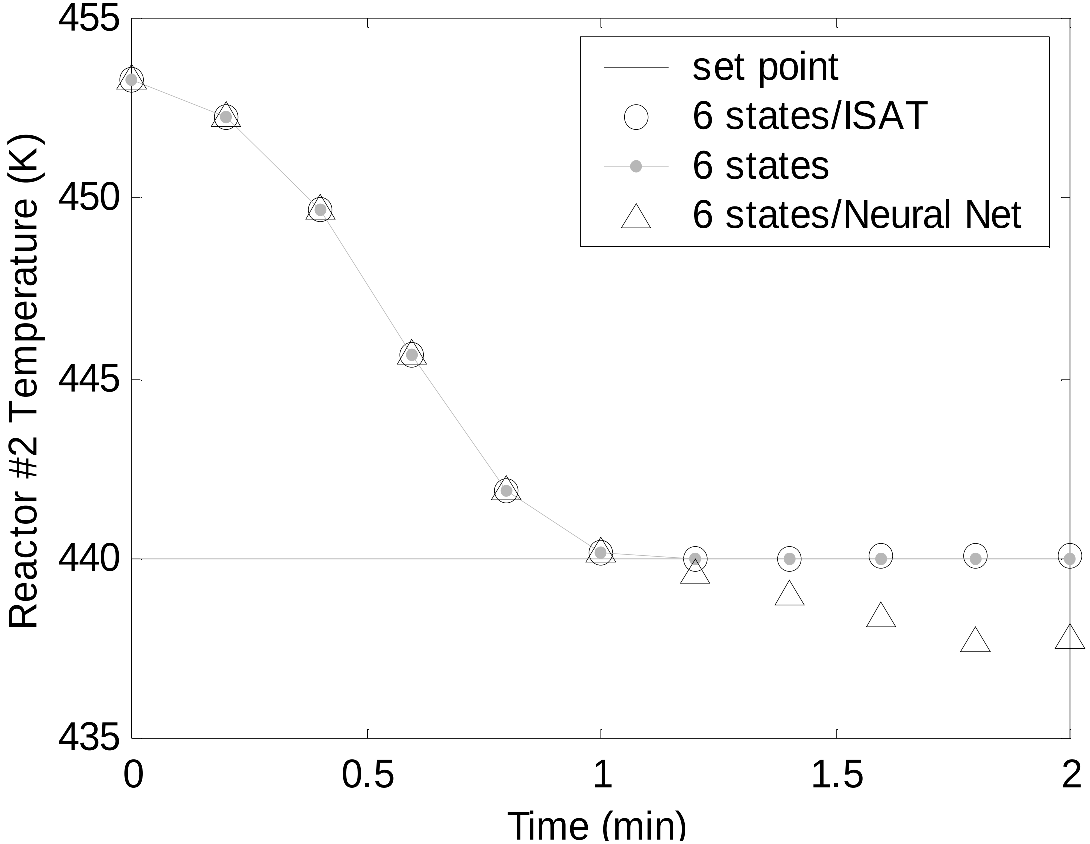


Fig. 11. Large closed loop set point change outside of the training domain.

Summary and Conclusions

- Combined approach includes model reduction followed by computational reduction

- ISAT is a storage and retrieval method

- With a 32 state binary distillation, the CPU time for NMPC is reduced by 85 times

Summary and Conclusions

- ISAT indicates when the retrieval is outside of the storage domain

- ISAT incorporates automatic error control to avoid extrapolation errors